

The charge $q(t)$ is related to the instantaneous membrane voltage $V_m(t)$ and membrane capacitance C_m by the relationship $q = C_m \cdot V_m$. Thus the capacitive current can be rewritten as

$$I_C = C_m \frac{dV_m}{dt} \dots \dots \dots (2)$$

In the Hodgkin-Huxley model of the squid axon, the ionic current I_{ion} is subdivided into three distinct components, a sodium current I_{Na} , a potassium current I_K , and a small leakage current I_L that is primarily carried by chloride ions. So the differential equation is given by

$$C_m \frac{dV_m}{dt} + I_{ion} = I_{app1} \dots \dots \dots (3)$$

Where I_{app1} is an externally applied current, such as might be introduced through an intracellular electrode.

The individual ionic currents I_{Na} , I_K and I_L represent the macroscopic currents flowing through a large population of individual ion channels. In HH-style models, the macroscopic current is assumed to be related to the membrane voltage through an Ohm's law relationship of the form $V=IR$ [5]. In many cases Ohm's law described by $I=GV$. Where G is the conductance and given by $G=1/R$. So the total ionic current I_{ion} is the algebraic sum of the individual contributions from all participating channel types found in the cell membrane is given by (46)

$$I_{ion} = \sum I_k = \sum G_k(V_m - E_k) \dots \dots \dots (4)$$

This expands to the following expression for the Hodgkin-Huxley model of the squid axon:

$$I_{ion} = G_{Na}(V_m - E_{Na}) + G_k(V_m - E_k) + G_L(V_m - E_L) \dots \dots \dots (5)$$

In general, the conductances are not constant values, but they can depend on other factors like membrane voltage or the intracellular calcium concentration. In order to explain their experimental data, Hodgkin and Huxley postulated that G_{Na} and G_K were voltage-dependent quantities, whereas the leakage current G_L was taken to be constant. Although Hodgkin and Huxley did not describe about the properties of individual membrane channels when they developed their model, it will be convenient for us to describe the voltage-dependent aspects of their model in those terms [5].

The FitzHugh–Nagumo model

One of the simplest single cell models is what is now called the FitzHugh–Nagumo (FHN) model. The model was originally developed as simplification of the Hodgkin–Huxley model by

FitzHugh in 1961. The behaviour of a neuron after stimulation by an external input current is expressed by equation (6) and (7) [6] and expressed in the equivalent circuit by Nagumo in 1962 [7].

$$\frac{dv}{dt} = v - \frac{v^3}{3} - w + I_{st} \dots \dots \dots (6)$$

$$\tau \frac{dw}{dt} = v + a - bw \dots \dots \dots (7)$$

If the external stimulus I_{ext} current I_{st} exceeds a certain threshold value, the system will exhibit a regular excitation. Where v is membrane voltage v and w is a linear recovery variable and a, b, τ are model parameters. An example of values for these parameters is given in Table 1. These parameters may be modified to model different cell types (Figure 1) [6].

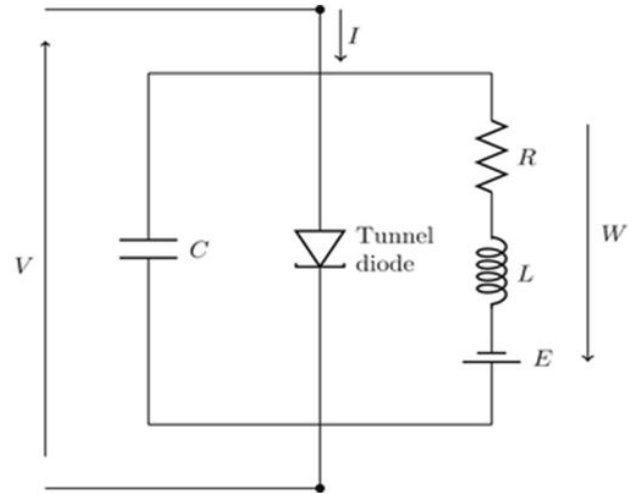


Figure 1. Circuit diagram of the tunnel diode FitzHugh–Nagumo model.

The motivation for the FHN model was to isolate conceptually the essential mathematical properties of excitation and propagation from the electrochemical properties of sodium and potassium ion flow [6].

Table 1. Parameters value of simple FitzHugh–Nagumo model [6].

Parameter	Value
I_{st}	0.32
a	0.7
b	0.8
τ	12.5

Results and Discussion

Solution of equation (6) and (7) are obtained using the parameters of Table 1. It has been observe that every parameter

Parameters estimation of Fitzhugh-Nagumo model

has a threshold value. Below or above this threshold regular excitation is occurred. Threshold label of I_{st} identified in this present studied is 0.324. Threshold value of a , b , τ are 0.69, 0.79 and 14.4 respectively. The results are shown graphically on Figures 2 to 5.

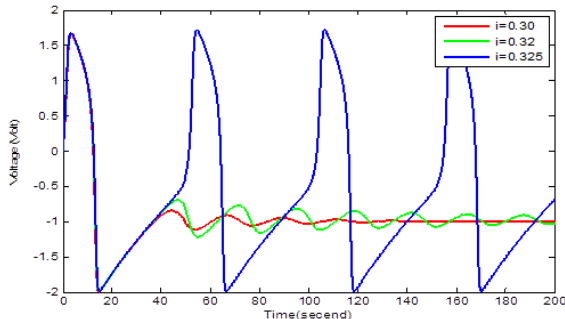


Figure 2. Graph of membrane voltage for $a=0.7$, $b=0.8$, $\tau=12.5$ and $I = 0.3, 0.32$ and 0.325 .

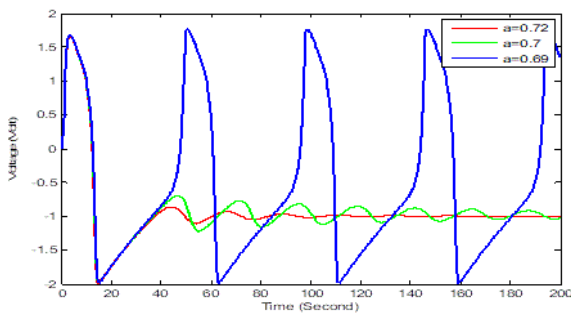


Figure 3. Graph of membrane voltage for $i=0.32$, $b=0.8$, $\tau=12.5$ and $a = 0.72, 0.7$ and 0.69 .

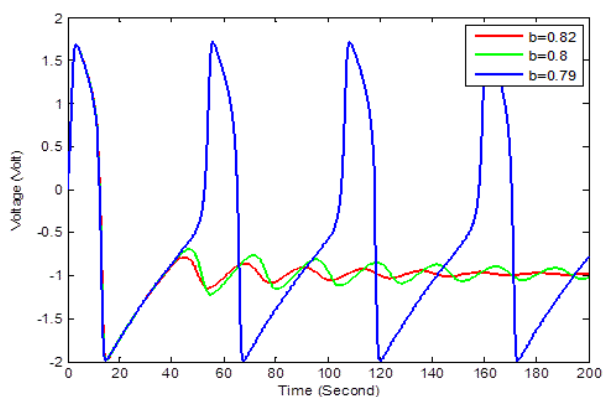


Figure 4. Graph of membrane voltage for $i=0.32$, $a=0.7$, $\tau=12.5$ and $b = 0.82, 0.8$ and 0.79 .

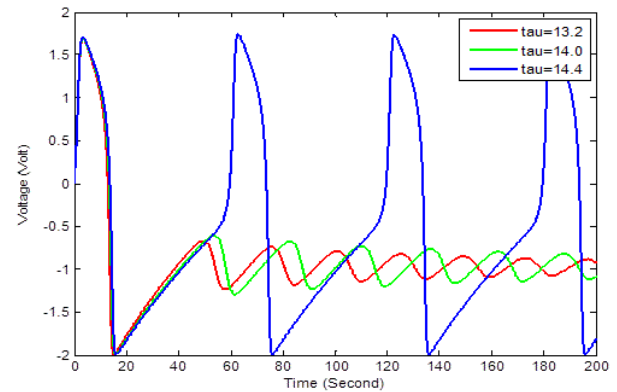


Figure 5. Graph of membrane voltage for $i=0.32$, $a=0.7$, $b=0.8$ and $\tau = 13.2, 14.0$ and 14.4 .

Conclusion

In this article, simulation of FitzHugh-Nagumo model is made. The parameters of FitzHugh-Nagumo model for regular excitation are studied. Simulation results are carried using MATLAB. The FHN model is simple to implement and computationally inexpensive but it is limited in terms of the physiological accuracy. Many attempts have been made to modify the FHN model to make it more physiologically accurate while retaining the simplicity of the original model.

References

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*Correspondence to

Iffat Ara

Pabna University of Science and Technology

Pabna

Bangladesh